Technical Notes

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Tangential Velocity and Static Pressure Distributions in Vortex Chambers

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Nomenclature

P = static pressure
P_a = ambient pressure
R_E = radius of the exhaust
R_0 = radius of the chamber
r = radial distance
v_r, v_θ = radial and tangential velocity components, respectively
ρ = fluid density
ϕ = angle between the total velocity vector and the tangential velocity component at the inlet

Subscript

in = properties evaluated on the inlet plane

Introduction

THE vortex chamber flow is known to be dominated by a large tangential velocity component which resembles the Rankine's velocity distribution with a viscous smoothening present in the vortex mode transition region.[1,2] Due to the strong swirling motion of the fluid inside the chamber, the radial static pressure distribution is arranged in such a manner as to balance the local centrifugal force. Recently, it was experimentally demonstrated that the dimensionless tangential velocity and static pressure distributions depend greatly on the geometrical parameters of the vortex chamber, and do not depend strongly on the inlet volumetric flowrate or the axial distance from the lower plate.[1,3] The latter conclusion is true for most of the vortex chamber flow except in the Ekman's boundary layers, at the top and bottom plates, where the value of the tangential velocity is first expected to peak,[4,5] and then reduce to zero.

In the present Note, Oseen's vortex is used to approximate the radial distribution of the tangential velocity and to calculate the static pressure in a vortex chamber. The results are compared with the experiment and formulas which are based on velocity profiles with continuous first derivatives as found in the technical literature.

Analysis

The experiments of the present study were conducted in Concordia's isothermal cyclone chamber test facility. For the velocity and pressure mesurements in the United Sensor's five-hole pilot probe DA-K25-24-F22-CD was used. The test facility and the experimental procedure are described in detail elsewhere.[2,4]

The observed tangential velocity distribution, depicted in Fig. 1, suggests a similar behavior of the velocity vs the radius with the theoretical results of Oseen and Hamel,[6]

$$\bar{v}_\theta = (1/\bar{r})(1 - e^{-a\bar{r}^2}) \tag{1}$$

where $\bar{v}_\theta = v_\theta / v_{\theta \mathrm{in}}$, $\bar{r} = r/R_0$, and the dimensionless time $4\nu t/R_0^2$ which appears in the original equation has been replaced by the constant a. The value of a is calculated to best fit Eq. (1) to the experimental data. This is achieved utilizing the least-squares method.

From Fig. 1 it can be seen that the best fit of Eq. (1) approximates well the observed velocity distributions. It is also evident from the same figure that the theoretical formula given by Basina et al.[7] underestimates the velocity for all three contraction ratios (R_E/R_0). The empirical formula for $\bar{v}_\theta$ given by Baluev and Troyankin[8] is not shown in Fig. 1, since their formulas give significantly lower values compared to the present experimental results. There is also no evidence of a strong saddle-like behavior of the tangential velocity near the circumferential wall, as was the case in the experiments of Baluev and Troyankin.[8] The latter phenomenon is also absent in the experiments of Reydon and Gauvin,[2] Ogawa,[9] and Najim et al.[10]

Since the static pressure inside the chamber is arranged in such a manner as to mainly balance the dominating local centrifugal forces, the radial momentum equation simplifies to

$$\frac{\mathrm{d}\bar{P}}{\mathrm{d}\bar{r}} = \frac{\bar{v}_\theta^2}{\bar{r}} \tag{2}$$

where $\bar{P} = P/\rho\, v_{\theta \mathrm{in}}^2$. Integration of Eq. (2) yields

$$\begin{aligned}\overline{\Delta P} = \overline{\Delta P}(\bar{r} = 1.0) - \cot^2\phi \Big\{ \left(\frac{1}{\bar{r}^2} - 1 \right) \\ + \bar{e}^{a}(2 - \bar{e}^{a}) - \frac{\bar{e}^{a\bar{r}^2}}{\bar{r}^2}(2 - \bar{e}^{a\bar{r}^2}) - 2a[E_i(-a\bar{r}^2) \\ - E_i(-2a\bar{r}^2)] + 2a[E_i(-a) - E_i(-2a)] \Big\}\end{aligned} \tag{3}$$

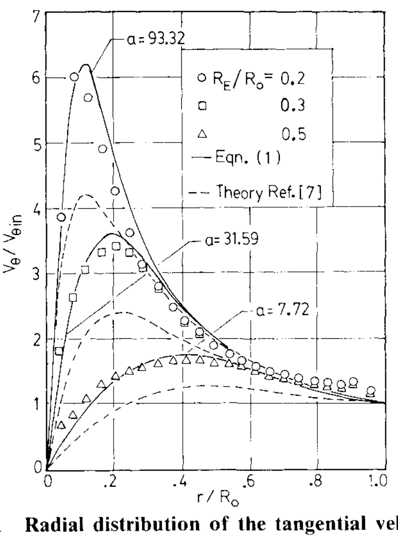

Fig. 1 Radial distribution of the tangential velocity.

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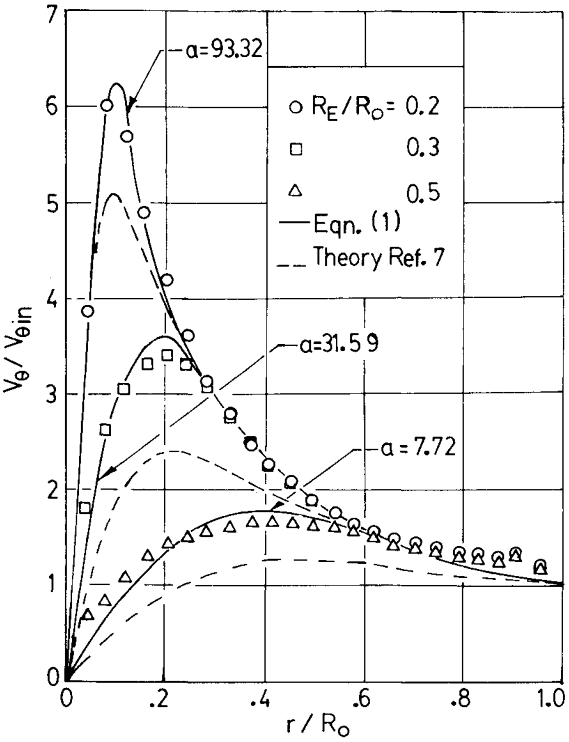

Fig. 2 Radial distribution of the static pressure.

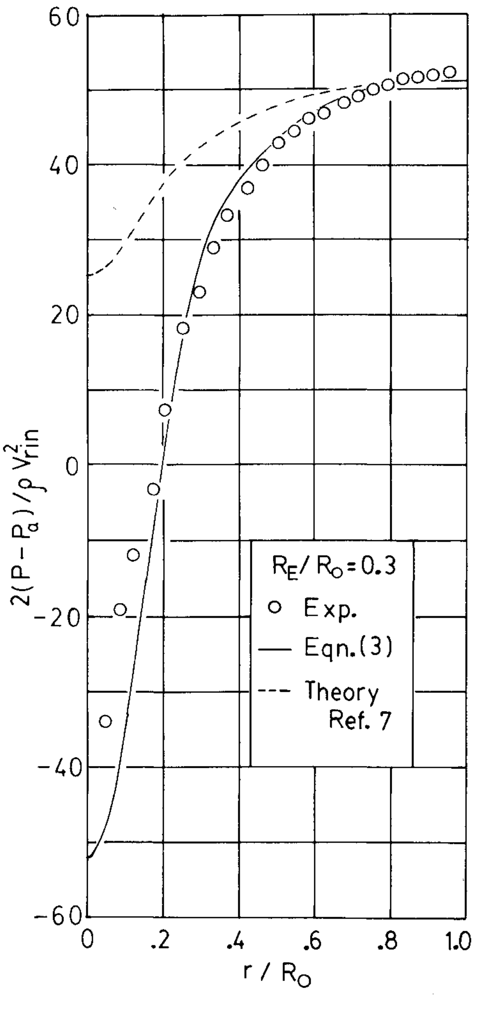

Fig. 3 Variation of parameter a with the contraction ratio.

where $E_i(-\xi) = -\int_\xi^\infty \exp(-\zeta)d\zeta$ is one of the exponential integrals, $\overline{\Delta P} = 2(P-P_a)/\rho v_{rin}^2$ ($v_{rin} = \tan\phi v_{\theta in}$), and $\overline{\Delta P}$ ($\bar r = 1.0$) is the dimensionless static pressure at the wall. The limiting value of $\overline{\Delta P}$ at the axis of rotation is

$$\lim_{\bar r \to 0} (\overline{\Delta P}) = \overline{\Delta P}(\bar r = 1.0) - \cot^2\phi\{2a\ell n 2 + \bar e^a(2 - \bar e^a) + 2a[E_i(-a) - E_i(-2a)]\}$$

The results of Eq. (3) are shown in Fig. 2. The static pressure, as expected, has a maximum value at the wall and decreases toward the axis of rotation. Inside the core (radius less than the radius of maximum tangential velocity) the pressure is seen to decrease drastically. In this region the agreement of Eq. (3) with the observed values of the pressure is relatively poor, since the visocus effects inside the core have been neglected. This disagreement was to be expected. The theoretical results calculated utilizing the best fit of the Basin et al.[7] equation for the tangential velocity, shows a significant disagreement for $\bar r < 0.7$.

From Fig. 1 it is evident that parameter a is a function of the contraction ratio. In Fig. 3, $\log_{10} a$ is plotted for values of R_E/R_0 within the closed interval [0.2, 0.5] utilizing Lagragian polynomials. From previous work, it is clear that the core size (therefore, a) depends also on the relative inlet swirl number. For this reason the results depicted in Fig. 3 must only be used for chambers which are geometrically similar to the typical vortex chamber presented in Refs. 1 and 3.

Conclusions

Experimental results and curve fitting equations for the tangential velocity and static pressure distributions inside a vortex chamber were presented. The formulas given in the literature were found to underestimate the velocity and overestimate the static pressure. No saddle-like behavior of the tangential velocity component near the circumferential was found to take place.

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